

# Math 131A-1: Homework 1

Due: April 1, 2016

1. Send me an e-mail introducing yourself. Let me know if you like to be called something other than your registrar listing, and anything you think I should know about your background.
2. Read Sections 1-3 in Ross.
3. Do problems 1.4, 1.8, 1.11 in Ross.
4. Prove Bernoulli's inequality: For  $x \in \mathbb{R}$  with  $x > 0$ , and every  $n > 1$ ,  $(1 + x)^n > 1 + nx$ .
5. Incorrect Inductions
  - (a) Consider the following inductive "proof" that all horses are the same color. We will show that any set of  $n$  horses have the same color. The base case is trivial, since any set consisting of a single horse has only one color. Now suppose that all sets of  $n - 1$  horses have only one color. Then if  $A = \{x_1, \dots, x_n\}$  is a set of  $n$  horses, consider the subsets  $A_1 = \{x_1, \dots, x_{n-1}\}$  and  $A_2 = \{x_2, \dots, x_n\}$ . Since each of  $A_1$  and  $A_2$  contain  $n - 1$  horses, all horses in  $A_1$  must be the same color and all horses in  $A_2$  must be the same color. And these sets overlap, so in fact all horses in  $A$  must be the same color. Therefore there is no horse of a different color!

Explain why this is not a valid inductive proof.

- (b) Consider the following inductive "proof" that all natural numbers are interesting. To begin with, the first case  $n = 1$  is clearly satisfied, since 1 is a very interesting number. Next, suppose there are uninteresting natural numbers. Then there must be a smallest such number, call it  $n$ . But  $n$  is the smallest uninteresting natural number, which is clearly an interesting thing to be! Therefore there aren't any uninteresting natural numbers.

Explain, in words, why this isn't a valid mathematical proof.